Fri Sep 28, 2012

SCORE: ___ / 25 POINTS

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: 3 / 3 POINTS**

Given sets A and B, the relation, R, with domain A and co-domain B, is a function if and only if,
① For every element $x \in A$, there is an element $y \in B$ such that $(x,y) \in R$.
② For every element $x \in A$, and elements $y \in B$ and $z \in B$, if $(x,y) \in R$ and $(x,z) \in R$ then $y = z$

Let $F = \{1, 3, 4\}$. 1 9 16 **SCORE: 6 / 6 POINTS**

Let $G = \{0, 2, 5\}$. 0 4 25

Let K be the relation from F to G defined by xKy if and only if $x^2 + y^2 = 5m$ for some integer m.

[a] Write K in set roster notation.

$1^2+0=1$ $1+2^2=5$ $1+5^2=26$ $3^2+0=9$ $3^2+2^2=13$ $3^2+5^2=34$
$4^2+2^2=20$ $4^2+5^2=41$

$K = \{(1,2), (4,2)\}$

$1\frac{1}{2}$ $1\frac{1}{2}$

[b] Is K a function? Why or why not?

No, $3 \in F$, yet there exists no corresponding element, y, in G for which $3Ky$ is true, therefore K is not a function.

$1\frac{1}{2}$

[c] If $H = \{b, c\}$, write $H \times G$ in set roster notation.

$\{(b,0), (b,2), (b,5), (c,0), (c,2), (c,5)\}$

$1\frac{1}{2}$

MULTIPLE CHOICE: Which of the following statements are true? **SCORE: 2 / 2 POINTS**

[1] $\{x\} \in \{\{x\}, y, z\}$ T
[2] $\{x\} \subseteq \{\{x\}, y, z\}$ F
[3] $\{z\} \subseteq \{\{x\}, y, z\}$ T

(a) none of the above (b) all of the above (c) only [1] (d) only [2]
(e) only [3] (f) only [1] and [2] (g) only [1] and [3] (h) only [2] and [3]

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Fill in the blanks for the following **formal definitions**. Use proper mathematical notation. **SCORE: ___ / 3 POINTS**

[a] The Cartesian product of sets P and Q is

$P \times Q =$ {(x,y) | x∈P AND y∈Q} $1\frac{1}{2}$

[b] Given sets P and Q, Q is a subset of P (or $Q \subseteq P$) if and only if

for all x∈Q, x∈P. *

If $W = \{0, 1, 2, 3, 4, 5\}$ and $Y = \{b, c, d, e, f, g, h\}$, **SCORE: 1 / 1 POINTS**
how many elements are in the Cartesian product of Y and W ?

7×6 = $\boxed{\text{42 elements}}$ 1

Classify each statement as Universal Existential (**UE**), Existential Universal (**EU**) or Universal Conditional (**UC**). **SCORE: 1 / 2 POINTS**

[a] Functions which have inverses must be one-to-one. UC 1

[b] There is a reciprocal for every natural number. EU

Let $A = \{x \in Z^* \mid -2 < x \leq 1\}$. 0, 1 * **SCORE: ___ / 5 POINTS**
Let $B = \{x \in Z \mid x^2 < 3\}$. -2, -1, 0, 1, 2 *
Let $C = \{0, 1\}$.

Are the following statements true or false ? Explain **very briefly** your answers. (No points if no explanation given.)

[a] A is a **proper** subset of C

FALSE. If A is a proper subset of C, then there must exist an element in C that does not exist in A.
There are no elements in C that do not exist in A.
Therefore, A is not a proper subset of C. 1

[b] $B = C$

FALSE, If B = C, they have to only have the same elements. B contains several elements that don't exist in C, therefore B ≠ C. *

Rewrite the following statement using the formal existential universal structure mentioned in the lecture notes. **SCORE: ___ / 3 POINTS**
NOTE: The answer requires 2 variables.
You may use algebra and/or symbolic set notation where appropriate.

"One of the instructors can teach every math class."

There exists an instructor, i, such that i can teach every math class, c.

1 *